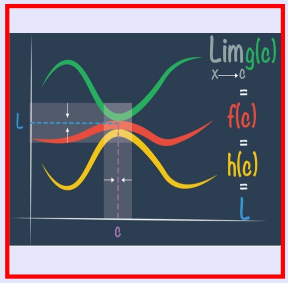
Math 261

Spring 2022

Lecture 2



Class QZ 1:

Use quadratic formula to Solve $3x^2-5x+2=0$.

Final Ans in a Solution Set. $ax^2+bx+c=0$

$a=3$ $b^2-4ac=(-5)^2-4(3)(2)=25-24=\boxed{1}$ ✓

$b=-5$

$c=2$

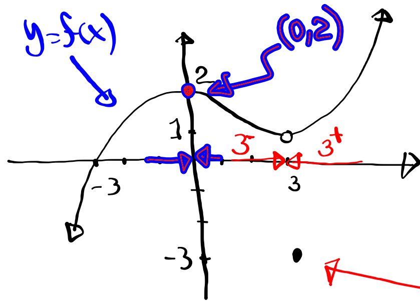
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-5)\pm\sqrt{1}}{2(3)}=\frac{5\pm1}{6}$$

$x=\frac{5+1}{6}=\frac{6}{6}=1$ ✓

$x=\frac{5-1}{6}=\frac{4}{6}=\frac{2}{3}$ ✓

$\left\{\frac{2}{3},1\right\}$

Consider the graph below



Domain $(-\infty,\infty)$

Range $(-\infty,\infty)$

x-Int $(-3,0)$

Y-Int $(0,2)$

as $x \to 3^+ \Rightarrow f(x) \to 1$

as $x \to 3^- \Rightarrow f(x) \to 1$

$\Rightarrow$ as $x \to 3 \Rightarrow f(x) \to 1$

Find $f(3)$

$f(3) = -3$

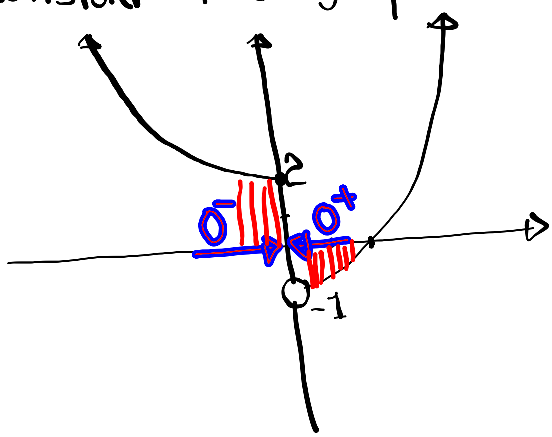
as $x \to 0^+ \Rightarrow f(x) \to 2$

as $x \to 0^- \Rightarrow f(x) \to 2$

$\Rightarrow$ as $x \to 0 \Rightarrow f(x) \to 2$

Find $f(0) = 2$

Consider the graph below



1) Domain $(-\infty,\infty)$

2) Range $(-1,\infty)$

3) x-Int $(2,0)$

4) Y-Int $(0,2)$

5) as $x \to 0^+ \Rightarrow f(x) \to -1$

6) as $x \to 0^- \Rightarrow f(x) \to 2$

7) $f(0) = 2$

8) as $x \to 0 \Rightarrow f(x) \to$ D.N.E.

Introduction to limits:

$\lim_{x \to a^+} f(x)$ "limit of $f(x)$ as x approches a from the right"

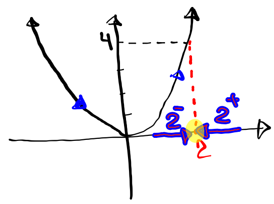
$\lim_{x \to a^-} f(x)$ "limit of $f(x)$ as x approches a from the left"

If $\lim_{x \to a^+} f(x) = L_1$ and $\lim_{x \to a^-} f(x) = L_2$

when $L_1 = L_2$ thim $\lim_{x \to a} f(x) = L$

where $L = L_1 = L_2$.

Consider $f(x) = x^2$



Domain $(-\infty, \infty)$

Range $[0, \infty)$

x-Int $\to (0,0)$

Y-Int $\to (0,0)$

Decreasing $(-\infty, 0)$

Min. Point $(0,0)$

Increasing $(0, \infty)$

No max. Point

$\lim_{x \to 2^+} f(x) = 4$, $\lim_{x \to 2^-} f(x) = 4$ So $\boxed{\lim_{x \to 2} f(x) = 4}$ ✓

$f(2) = 2^2 = \boxed{4}$ ✓

$\boxed{\text{If } \lim_{x \to a} f(x) = f(a), \text{ then } f(x) \text{ is continuous at } x = a.}$

$f(2.001) = 2.001^2 \approx 4.000\ldots$
$x \to 2^+$

$f(1.999) = 1.999^2 \approx 3.996\ldots$
$x \to 2^-$

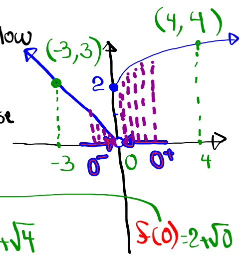
$\Rightarrow \to 4$

February 9, 2022

Consider the function below

$$f(x) = \begin{cases} -x & \text{if } x<0 \\ 2+\sqrt{x} & \text{if } x \geq 0 \end{cases}$$

Piece-wise



$f(-3) = -(-3) = \boxed{3}$

$f(4) = 2+\sqrt{4} = 2+2 = \boxed{4}$

$f(0) = 2+\sqrt{0} = \boxed{2}$

Find

$\lim_{x\to 0^-} f(x) = \boxed{0}$

$\lim_{x\to 0^+} f(x) = \boxed{2}$

$\lim_{x\to 0} f(x) =$ D.N.E.

$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

Is $f(x)$ continuous at $x=0$? explain

NO, $\underbrace{\text{Gap, Jump, hole}}_{\text{discontinuous}}$

$\lim_{x\to 0} f(x) \neq f(0)$

D.N.E. 2

How to find $\lim_{x\to a} f(x)$:

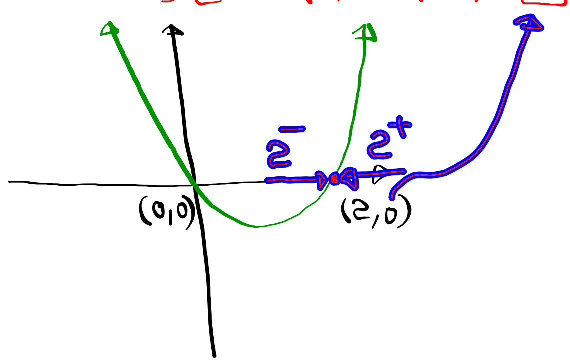
1) Evaluate $f(a)$, If you have an answer, that is the limit value.

$\lim_{x\to 2} (x^2-2x) = f(2)$

$f(x)$

a

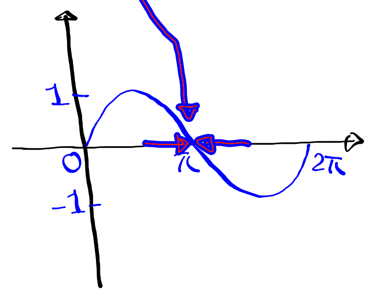
$= 2^2-2(2) = 4-4 = \boxed{0}$



Evaluate $\lim_{x \to \pi} \sin x = \sin \pi = \boxed{0}$

a

f(x)



Find $\lim_{x \to 1} \cos(\pi x) = \cos(\pi \cdot 1)$

$= \cos \pi$

$= \boxed{-1}$

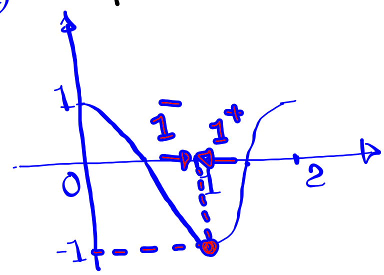
$\cos x$

$0 \le x \le 2\pi$

$0 \le \pi x \le 2\pi$

Divide by π

$0 \le x \le 2$



Find $\lim_{x \to 2} \dfrac{x^2-4}{x-2}$

First $f(x) = \dfrac{x^2-4}{x-2}$, $a = 2$ $\Rightarrow f(2) = \dfrac{2^2-4}{2-2} = \dfrac{4-4}{2-2} = \dfrac{0}{0}$

Rational Function

Indeterminate

Try to simplify

$f(x) = \dfrac{x^2-4}{x-2} = \dfrac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = x+2$

$\lim_{x \to 2} \dfrac{x^2-4}{x-2} = \lim_{x \to 2} (x+2) = 2+2 = \boxed{4}$

$\underset{2^+}{f(2.001)} = \dfrac{(2.001)^2-4}{2.001-2} \approx 4$

$\underset{2^-}{f(1.999)} = \dfrac{(1.999)^2-4}{1.999-2}$

≈ 4

Evaluate $\lim_{x \to 3} \frac{x^3 - 27}{x^2 + 2x - 15} = \frac{3^3 - 27}{3^2 + 2(3) - 15} = \frac{0}{0}$ I.F.

$A^3 - B^3 =$
$(A-B)(A^2 + AB + B^2)$

$$\lim_{x \to 3} \frac{x^3 - 27}{x^2 + 2x - 15} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x^2 + 3x + 9)}{(x+5)\cancel{(x-3)}}$$

$$= \lim_{x \to 3} \frac{x^2 + 3x + 9}{x + 5} = \frac{3^2 + 3(3) + 9}{3 + 5}$$

$$= \boxed{\frac{27}{8}} = \boxed{3.375}$$

Find $\lim_{x \to 1} \frac{x-1}{\sqrt{x} - 1} = \frac{1-1}{\sqrt{1} - 1} = \frac{1-1}{1-1} = \frac{0}{0}$ I.F.

when dealing with radicals, we can try to rationlize

Conjugate of $\sqrt{x} - 1$

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x} - 1} = \lim_{x \to 1} \frac{(x-1) \cdot (\sqrt{x} + 1)}{(\sqrt{x} - 1) \cdot (\sqrt{x} + 1)}$$

$$= \lim_{x \to 1} \frac{(x-1)(\sqrt{x} + 1)}{(\sqrt{x})^2 - (1)^2}$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(\sqrt{x} + 1)}{\cancel{x - 1}} = \lim_{x \to 1} (\sqrt{x} + 1)$$

$$= \sqrt{1} + 1 = \boxed{2}$$

Evaluate

1) $\lim_{x\to 5}\sqrt{x^3-3x-10} = \sqrt{5^3-3(5)-10} = \sqrt{125-15-10} = \sqrt{100} = \boxed{10}$

2) $\lim_{x\to 3}\dfrac{x^2-2x}{x+1} = \dfrac{3^2-2(3)}{3+1} = \dfrac{9-6}{4} = \boxed{\dfrac{3}{4}}$

3) $\lim_{x\to -1}\dfrac{x^2+6x+5}{x^2-3x-4} = \dfrac{(-1)^2+6(-1)+5}{(-1)^2-3(-1)-4} = \dfrac{1-6+5}{1+3-4} = \dfrac{0}{0}$ I.F.

$\lim_{x\to -1}\dfrac{x^2+6x+5}{x^2-3x-4} = \lim_{x\to -1}\dfrac{\cancel{(x+1)}(x+5)}{(x-4)\cancel{(x+1)}} = \lim_{x\to -1}\dfrac{x+5}{x-4}$

$= \dfrac{-1+5}{-1-4} = \dfrac{4}{-5} = \boxed{-\dfrac{4}{5}}$

4) $\lim_{x\to 9}\dfrac{\sqrt{x}-3}{x-9} = \dfrac{\sqrt{9}-3}{9-9} = \dfrac{3-3}{9-9} = \dfrac{0}{0}$ I.F.

Hint: Rationalize the numerator.

$\lim_{x\to 9}\dfrac{\sqrt{x}-3}{x-9} = \lim_{x\to 9}\dfrac{\overbrace{(\sqrt{x}-3)(\sqrt{x}+3)}^{\cancel{x-9}}}{\cancel{(x-9)}(\sqrt{x}+3)} = \lim_{x\to 9}\dfrac{1}{\sqrt{x}+3}$

5) $\lim_{x\to 0}\dfrac{\sqrt{x+3}-\sqrt{3}}{x} = \dfrac{\sqrt{0+3}-\sqrt{3}}{0} = \dfrac{0}{0}$ I.F. $= \dfrac{1}{\sqrt{9}+3} = \boxed{\dfrac{1}{6}}$

$\lim_{x\to 0}\dfrac{\sqrt{x+3}-\sqrt{3}}{x} = \lim_{x\to 0}\dfrac{\overbrace{(\sqrt{x+3}-\sqrt{3})(\sqrt{x+3}+\sqrt{3})}^{A^2-B^2}}{x(\sqrt{x+3}+\sqrt{3})}$

$= \lim_{x\to 0}\dfrac{x+\cancel{3}\ \cancel{-3}}{x(\sqrt{x+3}+\sqrt{3})}$

$= \lim_{x\to 0}\dfrac{\cancel{x}}{\cancel{x}(\sqrt{x+3}+\sqrt{3})} = \lim_{x\to 0}\dfrac{1}{\sqrt{x+3}+\sqrt{3}}$

$= \dfrac{1}{2\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \boxed{\dfrac{\sqrt{3}}{6}}$ ✓

Evaluate $\lim_{x \to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x-2} = \dfrac{\frac{1}{2} - \frac{1}{2}}{2-2} = \dfrac{0}{0}$ I.F.

Use LCD to Simplify LCD $= 2x$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} = \lim_{x \to 2} \frac{2\cancel{x} \cdot \frac{1}{\cancel{x}} - \cancel{2}x \cdot \frac{1}{\cancel{2}}}{2x(x-2)}$$

$$= \lim_{x \to 2} \frac{2-x}{2x(x-2)} = \lim_{x \to 2} \frac{-1\cancel{(x-2)}}{2x\cancel{(x-2)}}$$

$$= \lim_{x \to 2} \frac{-1}{2x} = \frac{-1}{2(2)} = \boxed{\frac{-1}{4}}$$

Evaluate

$$\lim_{x \to 1} \frac{x^3 + x^2 - 5x + 3}{x^3 - 3x + 2} = \frac{1^3 + 1^2 - 5(1) + 3}{1^3 - 3(1) + 2} = \frac{0}{0}$$

I.F.

Synthetic Division

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -5 & 3 \\ & & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & 0 \end{array} \qquad \begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

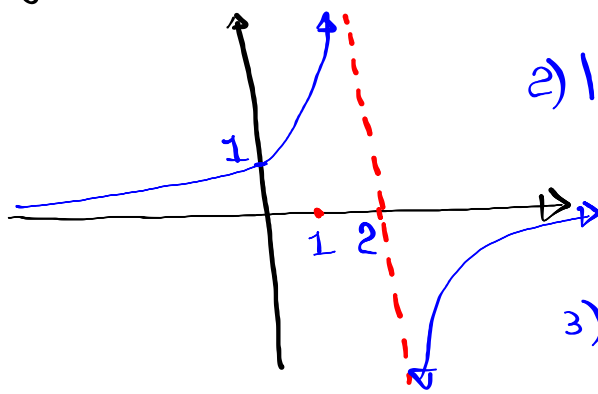
$$\lim_{x \to 1} \frac{x^3 + x^2 - 5x + 3}{x^3 - 3x + 2} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2 + 2x - 3)}{\cancel{(x-1)}(x^2 + x - 2)}$$

$$= \lim_{x \to 1} \frac{x^2 + 2x - 3}{x^2 + x - 2} = \frac{0}{0} \text{ I.F.}$$

$$= \lim_{x \to 1} \frac{(x+3)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}} = \lim_{x \to 1} \frac{x+3}{x+2} = \boxed{\frac{4}{3}}$$

Class QZ 2

Consider the graph of $y = f(x)$ below



1) $\lim_{x \to 2^-} f(x) = \boxed{\infty}$

2) $\lim_{x \to 2^+} f(x) = \boxed{-\infty}$

3) $\lim_{x \to 0} f(x) = \boxed{1}$